# Encontro Nacional da Sociedade Portuguesa de Matemática

## 25 a 28 de Junho de 2008

## ISEC

### Física-Matemática

Org. Marco Mackaay

26 de Junho, 5ª feira, 11h-12h30

- Aleksandar Mikovic: *Spin Foam Perturbation Theory for Three-Dimensional Quantum Gravity*
- Vladimir Dragovic: *Integrable Billiards, Hyperelliptic Jacobians and Pencils of Quadrics*
- João Martins: *Two-Dimensional holonomy*

27 de Junho, 6ª feira, 11h-12h30m

- Roger Picken: *A quantum Goldman bracket in 2+1 quantum gravity*
- Jean-Claude Zambrini: *Geometry of stochastic differential equations*
- Paulo Pinto: *Operator Algebras and Mathematical Physics*

27 de Junho, 6ª feira, 14h-15h30m

- Ricardo Schiappa: *Instantons and Matrix Models*
- Atle Hahn: *From the Chern-Simons path integral to the Reshetikhin-Turaev invariant*
- José Mourão: *Geometric Quantization and Tropical Amoebas on Toric Varieties*

28 de Junho, Sábado, 11h-12h30

- Nenad Manojlovic: *Izergin-Korepin type R-matrices in quantum solvable systems*
- Diogo Pinheiro: *Dynamics of two interacting charges in a uniform magnetic field*
- Pedro Vaz: *sl(N)-link homology using foams and the Kapustin-Li formula*

# Integrable Billiards, Hyperelliptic Jacobians and Pencils of Quadrics

**Vladimir Dragovic**

GFMUL

**Abstract**

We consider a family of confocal quadrics in $d$ dimensional space and a set $T$ of lines tangent to the subset of $d-1$ quadrics from the given family. The set $T$ is invariant with respect to the billiard dynamics generated by the confocal family. We construct and study an algebraic structure on $T$, which enables us to derive higher-dimensional and higher-genera generalizations of several well-known and classical results. Most of the results are presented in the paper arXiv:0710.3656 joint with M. Radnovic.

# From the Chern-Simons path integral to the Reshetikhin-Turaev invariant

**Atle Hahn**

GFMUL

**Abstract**

The study of the heuristic Chern-Simons path integral by E. Witten inspired (at least) two general approaches to quantum topology. Firstly, the perturbative approach based on the CS path integral in the Lorentz gauge and, secondly, the "quantum group approach"by Reshetikhin/Turaev. While for the first approach the relation to the CS path integral is obvious for the second approach it is not. In particular, it is not clear if/how one can derive the relevant R-matrices or quantum 6j-symbols directly from the CS path integral. In my talk, which summarizes the results of a recent preprint, I will sketch a strategy that should lead to a clarification of this issue in the special case where the base manifold is of product form. This strategy is based on the "torus gauge fixing"procedure introduced by Blau/Thompson for the study of the partition function of CS models. I will show that the formulas of Blau/Thompson can be generalized to Wilson lines and that the evaluation of the expectation values of these Wilson lines leads to the same state sum expressions in terms of which Turaev's shadow invariant is defined. Finally, I will sketch how one can obtain a rigorous realization of the path integral expressions appearing in this treatment.

# Izergin-Korepin type R-matrices in quantum solvable systems

**Nenad Manojlovic**

GFMUL and Universidade do Algarve

**Abstract**

Some algebraic structures related to the Izergin-Korepin type R-matrices are reviewed. Algebraic Bethe ansatz is then implemented for the corresponding quantum solvable systems.

# Two-Dimensional holonomy

**João Martins**[1]
CMUP

**Abstract**

Given a Lie crossed module $\mathcal{G}$ and a $\mathcal{G}$-2-bundle over a smooth manifold $M$, we recall the construction of $\mathcal{G}$-valued 2-dimensional holonomies. We use this framework to define Wilson surface observables for higher gauge theory.

[1]This is joint work with Roger Picken

# Spin Foam Perturbation Theory for Three-Dimensional Quantum Gravity

**Aleksandar Mikovic**

GFMUL and Universidade Lusófona

**Abstract**

We describe the spin foam perturbation theory for the three-dimensional Euclidean Quantum Gravity and analyze it in the dilute gas limit. We show that the Baez conjecture does not hold for arbitrary triangulations; however, it holds for a special class of triangulations which are based on the baricentric divisions of 3-manifold cubulations. In this case we calculate the partition function.

# Geometric Quantization and Tropical Amoebas on Toric Varieties

**José Mourão**[(a)], **Thomas Baier, Carlos Florentino and João P. Nunes**

[(a)]CAMGSD and Instituto Superior Técnico

**Abstract**

We consider toric deformations of complex structures, with degenerate limits of the holomorphic polarization corresponding to the toric Lagrangian fibration, in the sense of geometric quantization. This allows us to interpolate continuously between quantizations in the holomorphic and real polarizations and show that the monomial holomorphic sections of the prequantum bundle converge to Dirac delta distributions supported on Bohr-Sommerfeld fibers.

We use these families of toric metric degenerations to study the limit of compact hypersurface amoebas and show that they are described by tropical amoebas. We believe that our approach gives a different, complementary, perspective on the relation between complex algebraic geometry and tropical geometry. We also comment briefly on the relation of our results to some recent approaches to homological mirror symmetry.

# A quantum Goldman bracket in 2+1 quantum gravity

**Roger Picken**

CAMGSD and Instituto Superior Técnico

**Abstract**

This is a report on work with J.Nelson (Turin) concerning 2+1 quantum gravity on the torus with negative cosmological constant. In this approach quantum holonomy matrices are assigned to loops on the torus (using Witten's Chern-Simons perspective on 2+1 quantum gravity). This leads to a q-deformed representation of the fundamental group, where the matrices assigned to homotopic loops are related by phases coming from the signed area swept out by the homotopy. One can then obtain a quantum version of the Goldman bracket (a Poisson bracket for loops on surfaces, which is non-vanishing when the two loops intersect transversally, yielding loops which are rerouted at the intersection points). The area phases play an interesting role here, and are connected to a quantum version of Pick's theorem (a formula for the area of a polygon whose vertices lie on an integer lattice in the plane), as well as (I suspect) to parallel transport for non-abelian gerbes.

# Dynamics of two interacting charges in a uniform magnetic field

**Diogo Pinheiro**[1]

CMUP and FCT - Universidade Nova de Lisboa

**Abstract**

We study the interaction of two charges under the action of a uniform magnetic field.

We first look at the planar problem, i.e. the two charged particles move in $\mathbb{R}^2$. In this setting, we formulate the problem as a four degrees of freedom Hamiltonian system. We prove that this system can always be reduced to one with two degrees of freedom. Furthermore, we identify (distinct) sets in the space of parameters where:

a) the system is integrable;

b) the system is chaotic.

Based on the properties of the reduced system we provide a detailed qualitative description of the original four degrees of freedom system.

For the analysis of the spatial problem, we use similar techniques to reduce the dynamics and obtain a asymptotic description for the scattering map associated with this problem.

[1]This is joint work with R.S. MacKay (University of Warwick).

# Operator Algebras and Mathematical Physics

**Paulo Pinto**

CAMGSD and Instituto Superior Técnico

**Abstract**

Subfactor theory initiated by V. Jones in the 1980's is a natural framework to systematically classify modular invariant partition functions in RCFT, e.g. WZW SU(n) level $k$ and holomorphic orbifolds models. With this formalism, the highest weights reps or primary fields are replaced by braided endomorphisms on a fixed von Neumann factor $N$, and recaptures the underlying unitary representation of the modular group SL(2,Z) in RCFT. Then every subfactor $N \subseteq M$ yields a modular invariant with nice representations of the Verlinde fusion rules and graphs, empirically obtained before. We plan to provide the basics of this framework together with recent results illustrated in the (possibly $H^3$-twisted) quantum $S_3$ and $\mathbb{Z}_p$ finite groups. Finally we will yield a model for which no physical construction has been found yet.

# Instantons and Matrix Models

**Ricardo Schiappa**

CAMGSD and Instituto Superior Técnico

**Abstract**

Most quantum problems are solved using perturbation theory, which is known not to converge, and not to be Borel summable in many situations. The large order behavior of perturbation theory, and its divergence, is controlled by instantons, non-perturbative solutions to the Euclidean equations of motion. We shall study instantons and large-order behavior in the simplest possible examples, those of matrix models — 0-dimensional field theories. Interestingly enough, matrix models are also related to string theory and we shall see how to apply instanton calculus in string theory via matrix models. We will study the large order behavior both analytically, up to two loops, and numerically, in several cases of both pure matrix models and topological strings. Some of this information may also be derived in the double-scaling limit via the Painleve I equation, an integrable system which describes 2-dimensional quantum gravity. If time permits, we shall also cover applications to cases involving multi-instantons, and matrix models with several cuts in the complex plane.

# sl(N)-link homology using foams and the Kapustin-Li formula

**Pedro Vaz**

CAMGSD and Universidade do Algarve

**Abstract**

In joint work with M. Mackaay and M. Stosic [4], we define an almost topological construction of a rational link homology categorifying the sl(N)-link invariant. This construction uses foams which generalize the ones introduced by Khovanov in [1]. The evaluation of closed foams uses the Kapustin-Li formula, adapted to the context of foams by Khovanov and Rozansky [2]. Our link homology theory is equivalent to Khovanov and Rozansky's in [3]. In this talk I will present the topological aspects of this theory and show how to use the Kapustin-Li formula in order to evaluate the closed foams.

## Referências

[1] M. Khovanov, *sl(3) link homology*, Alg. Geom.Top. 4:1045-1081 (2004).

[2] M. Khovanov and L. Rozansky, *Topological Landau-Ginzburg models on the world-sheet foam*, Adv. Theor. Math. Phys., 11(2):233-260, (2007).

[3] M. Khovanov and L. Rozansky, *Matrix factorizations and link homology*, Fund. Math. 199(1):1-91 (2008).

[4] M. Mackaay, M. Stosic and P.Vaz, *sl(N)-link homology using foams and the Kapustin-Li formula*, arXiv:0708.2228 [math.GT].

# Geometry of stochastic differential equations

**Jean-Claude Zambrini**

GFMUL and Universidade de Lisboa

**Abstract**

We are going to describe various geometrical aspects of the theory of Stochastic Differential Equations linked, in particular, to the variational characterizations of their solutions,in the context of a general program of Stochastic Deformation of "classical"geometry.